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This test contains 4 questions on 3 pages.

A few **preliminary remarks**:

- Please answer questions 3 & 4 on another (double) sheet of paper than questions 1 & 2.
- Put your name and student number at the top of all sheets.
- Put your student card at the edge of the desk for checking by the assistants and show it when handing in your test.
- Add the units to the numbers calculated.

Question 1 (6 points): plane harmonic waves

A plane harmonic electromagnetic wave is specified (in SI units) by the following wave function:

$$\vec{\mathbf{E}} = \left(4\hat{\mathbf{i}} - 6\hat{\mathbf{j}}\right)\left(10^3\ \mathrm{V/m}\right)\ \cos\left[\left(3x+2y\right)\pi\times 10^7 - 1.26\times 10^{16}t\right]$$

with $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ the unit vectors along the x- and y-axis, respectively.

Questions:

a) Draw the direction in the x-y plane along which the electric field oscillates.

b) What is the scalar value of the amplitude of the electric field ?

c) What is the direction of propagation of the wave ? Indicate this direction in the drawing used in the answer of question a).

d) What is the wavelength of the wave ?

e) What is the frequency of the wave ?

Question 2 (4 points): Intensity and amplitude of an electromagnetic wave

Assume a laser beam (i.e. a harmonic electromagnetic wave) in vacuum with a power of 2000 W that is concentrated to a cross section of 10^{-6} cm^2.

Questions:

1. What is the irradiance of this beam ?
2. What is the amplitude of the electric field of the corresponding harmonic electromagnetic wave ?
3. What is the amplitude of the magnetic field of the corresponding harmonic electromagnetic wave ?

Give the physical units for all answers !

Extra information:

The size of the Poynting vector is given by: $S(t) = c^2\ \varepsilon_0\ E(t)\ B(t)$

with: c the speed of light in vacuum = 3×10^8 m/s

ε_0 the permittivity of the vacuum = 8.854×10^{-12} J/(V^2 m)

$E(t)$, $B(t)$ the size of the electric, magnetic field

Note: use a new (double) sheet of paper for questions 3 and 4.

Question 3 (4 points): Standing waves

The following expression represents a standing wave:

$$E = 150\ \sin\tfrac{1}{3}\pi x\ \cos 4\pi t$$

(x is the spatial coordinate, t is time)

Questions:

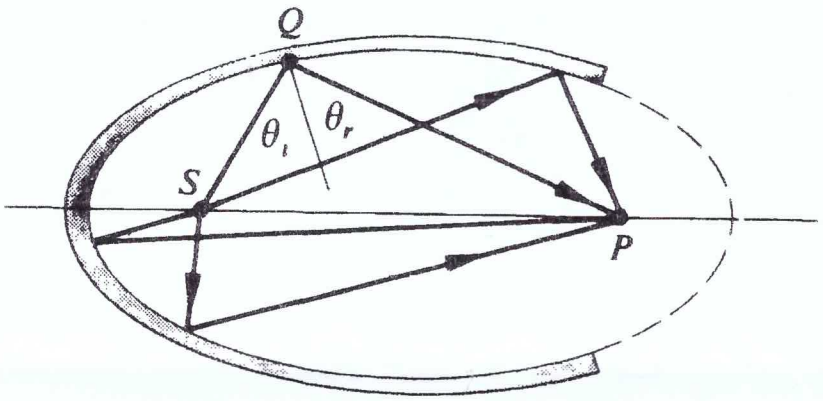
a) Why is this called a "standing" wave ?
b) Give 2 wave functions which, when superposed, generate the standing wave given above.

Question 4 (6 points): Fermat's principle

Fermat's principle allows to determine the manner in which light propagates.

a) What does the **original formulation** of Fermat's principle say about the path light will follow ? Give **two** equivalent versions. (No derivations are asked for, just state the principle.)

b) In some situations, Fermat's principle is not valid. In the lectures, an example of such a situation was discussed involving a source *S* and observation point *P* in the focii of an ellipsoid. The figure below should bring this situation to mind. For an ellipsoid, all paths SQP (with Q any point on the ellipsoid) have the same length.



Use this example to explain a situation in which Fermat's principle is not valid.

c) The existance of situations in which Fermat's principle is not valid has resulted in a **modern formulation** of Fermat's principle which is always valid. Give this modern formulation and explain its meaning.
